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The blister test – Transition from plate to membrane behaviour for an elastic material

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Abstract. The crack extension force has been calculated for the blister test when the blister is in transition from plate-like to membrane behaviour. Simple equations are given for the crack extension force that are accurate to better than 1%. The mode-mixity of interfacial fracture in the blister test is discussed and it is shown that the interfacial fracture energy is likely to increase with decrease in thickness of the film. In practical tests, the behaviour of a blister is likely to be nearer to a plate than a membrane.

1. Introduction

The blister test is a very convenient test to measure the adhesion energy of thin materials. In the normal blister test, pressure is introduced into a circular shaped pre-delamination between a thin material and a thick substrate and critical value of the pressure necessary to cause the delamination to grow is measured (see Figure 1). With the growth in interest in the mechanical behaviour of microelectronic devices, the blister test has become even more important.

Although Dannenberg (1961) is usually credited with the introduction of the blister test, his test is significantly different to today's blister test. Dannenberg used a grooved restraining plate to limit the deflection of the blister. He found that an oblong blister rather than a circular blister had a more stable debonding pattern. Using mercury as the pressurisation medium, the debonding of the restrained blister is stable. Dannenberg estimated the adhesion energy from the work of debonding. He realised that not all of the work went into the work of debonding and attempted to make allowances for the extra work by performing a dummy test on a clamped film. It was Williams (1969) who introduced the blister test as we now know it and applied fracture mechanics in the analysis two limiting conditions: a thin plate with small deflections and a very thick disk. In a later paper he considered an adhesive layer using a Winkler elastic foundation to model the adhesive layer and taking into consideration the energy stored in the adhesive layer (Williams, 1970). Williams and his co-workers also examined the transition from plate-like behaviour to thick-disk behaviour for small deflections using finite elements (Bennett et al., 1974). Provided the radius a, to thickness h, ratio of the delamination was greater than 10, the behaviour was plate-like.

Andrews and Stevenson (1978) calculated the crack extension force for thick plates with small deflections. Their limiting solution for an infinitely thick plate agrees with the result of Sneddon (1946).

Hinkley (1983) examined the case of very thin films which behave like membranes and showed that the crack extension force G, could be expressed in terms of the pressure q, and the maximum deflection w_0, by

$$G = \Phi q w_0. \tag{1}$$

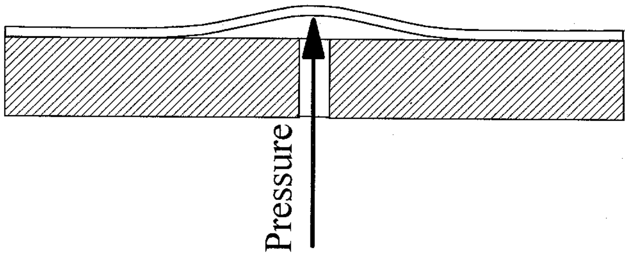

Figure 1. Schematic blister test.

Table 1. The crack extension force factor Φ_m, for a membrane blister.

ν	0.25	0.30	0.35	0.40	0.45	0.50
Φ_m	0.6531	0.6517	0.6503	0.6487	0.6472	0.6456

Hinkley based his calculation on the membrane solution of Hencky (1915), but his value of the crack extension force factor Φ, is incorrect. Hinkley's paper is the start of a long controversy over the correct value of the crack extension force for a blister behaving like a membrane. The various expressions that have been proposed for Φ are summarized by Briscoe and Panesar (1991). The membrane problem is not linear and the loss in potential energy of the pressure forces, due to the formation of the blister, is not twice the gain in strain energy in the membrane as it is in linear problems.[1] Some of the previous membrane analyses are incorrect because they have been treated as if the problem was linear.

The usual blister test is unstable and only a single result can be obtained from a test specimen. Wan and Mai (1995) have proposed an interesting modification to the blister test that makes it stable. Instead of simply pressurizing the blister, they sealed a small volume of gas containing the blister and 'pressurized' the blister by applying a vacuum to the other side. In effect their test is a constrained blister test similar to that of Dannenberg (1961). Wan and Mai (1995) also gave a correct derivation of the crack extension force for a membrane using the results of Hencky (1915).

Recently the other Williams (1997) has given a very comprehensive survey of the crack extension forces for peeling flexible membranes and blister tests. In his analysis of the blister test he found that Hencky (1915) had a small numerical error in his paper that affected the exact value of the crack extension force. The correct values of the crack extension force parameter Φ are given in Table 1. Although the deflections are large in the membrane solution given in Table 1, they are only large in the sense that $(\mathrm{d}w/\mathrm{d}r)^2$ is significant in the expression for the radial strain

$$\varepsilon_r = \frac{\mathrm{d}u}{\mathrm{d}r} + \frac{1}{2}\left(\frac{\mathrm{d}w}{\mathrm{d}r}\right)^2, \tag{2}$$

where u is the radial displacement, w the blister deflection and r the radial coordinate.

[1] For the membrane problem the loss of potential energy of the pressure forces is four times the gain in strain energy (Wan and Mai, 1995).

Williams has also considered very large deflections where $(\mathrm{d}u/\mathrm{d}r)^2$ cannot be neglected in the expression for the strain (Williams, (1997)). Assuming a hyper elastic solid, Williams (1997) has shown that even if the blister deflection is 1.3 times its radius a, that the crack extension force is only increased by 3%. Hence for all practical cases the values of the crack extension force factors given in Table 1 hold for very thin membranes.

The form of the crack extension force given in Equation 1 is general for all crack problems where the loading is by pressurisation of the entire crack area. It is an extremely useful form since the expression is independent of the crack size. In many applications the critical deflection of the blister is such that its behaviour is in the transition region from thin plate to membrane. For small displacements the crack extension force factor, Φ, is 0.5 and is independent of Poisson's ratio.

In a penetrating re-examination of the peel test Kinloch et al. (1996; 1994) have examined the boundary condition at the tip of the delamination. Because in analyses using beam or plate theory it is the deflection of the neutral axis that is specified, the built-in condition usually specified is not exact. It is the outer most fibre of the beam or plate that is actually built-in thus it is possible for the neutral axis to have a rotation and deflection at the tip of a delamination. The effect is most marked if the film is bonded to a substrate by an adhesive layer when the effect of the 'elastic foundation' is well known. However, even if a film is bonded directly to a substrate the film itself provides an effective 'elastic foundation' which is estimated to be half the thickness of the film (Kanninen, 1974; Williams, 1989). Kinloch et al. (1996; 1994) have shown that the effect of rotation at the tip of the delamination can have a very significant effect on the energy released for fracture in peel tests performed on thin films and it is necessary to examine whether this effect is also important in the blister test. Dimensional arguments show that for films directly bonded to a substrate without an adhesive layers the contribution to the crack extension force from energy stored in the 'elastic foundation' is of the order $qw_0(h/a)$ for small deflections and $qw_0(h/w_0)^2(h/a)$ for large deflections and hence for thin blisters the effect is negligible. If the film is bonded to a substrate by an adhesive layer h_a thick with a Young's modulus of E_a the contribution to the crack extension force from the 'elastic foundation' may be more significant if $(Eh_a/E_ah) < 1$ since the contribution for small deflections to the crack extension force is of the order $(Eh_a/E_ah)^{1/4}$ times that for films directly bonded to a substrate. The effect of an 'elastic foundation' is not considered in this paper.

The mode-mixity in interface fractures can be important because the fracture energy usually increases considerably when the mode II component becomes significant (Evans et al., 1990). The mode of fracture cannot be obtained from an energy calculation of the crack extension force. Hutchinson et al. (1992) have shown that the mode-mixity of the interface fracture of a thin blister formed by the buckling of a thin film under compressive residual stresses can be obtained from the membrane force and the bending moment at the edge of the blister. The shear stresses at the edge of the blister are of the order $(h/a)^3$ and are unimportant for small displacements of thin films because the bending and membrane stresses are of the order $(h/a)^2$. Hutchinson et al. (1992) first calculate the bending moment M, and the membrane force N, at the edge of the blister and then use the results of a previous paper (Suo and Hutchinson, 1990) to calculate the mode-mixity from these values. The same procedure for calculating the mode-mixity adopted by Hutchinson et al. (1992) has been used in this paper.

2. The crack extension force

In the transition from thin plate to membrane behaviour of blisters, the equations for large deflections of plates can be used (Timoshenko and Woinowsky-Krieger, 1959). These equations are most conveniently written in their non-dimensional form

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}\xi}\left[\frac{1}{\xi}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\frac{\mathrm{d}W}{\mathrm{d}\xi}\right)\right] - 12(1-\nu^2)N - 32Q = 0,\\
&\xi\frac{\mathrm{d}}{\mathrm{d}\xi}\left[N + \frac{\mathrm{d}}{\mathrm{d}\xi}(\xi N)\right] + \frac{1}{2}\left(\frac{\mathrm{d}W}{\mathrm{d}\xi}\right)^2 = 0,
\end{aligned} \tag{3}$$

where W is the deflection (w) of the blister non-dimensionalised by its thickness (h); N is the radial membrane stress non-dimensionalised by $E(h/a)^2$, and E is the Young's modulus; Q is the pressure (q) inside the blister non-dimensionalised by $16E/3(1-\nu^2)(h/a)^4$ and ν is Poisson's ratio; and ξ is the non-dimensionalised radius r/a. The boundary conditions at the edge of the blister $(r = a)$ are zero slope and zero radial displacement u. With this non-dimensional scheme, the deflection $W_{0,}$ at the centre of the blister is equal to Q for very small deflections. The most suitable large deflection solution for calculating the crack extension force G, is that of Way (1934). The non-dimensional slope, $\frac{\mathrm{d}W}{\mathrm{d}\xi}$, and the radial membrane stress, N, can be represented by the series:

$$\begin{aligned}
&\frac{\mathrm{d}W}{\mathrm{d}\xi} = -4Q\sum_{k=1,3,5}^{\infty} C_k\xi^k,\\
&N = \sum_{k=0,2,4}^{\infty} B_k\xi^k.
\end{aligned} \tag{4}$$

For very small displacements $C_1 = 1, C_2 = -1$ and all other coefficients are zero. Substituting these series into Equations (3) Way (1934) showed that all the coefficients could be solved sequentially in terms of C_1 and B_0. The values of C_1 and B_0 that satisfy the boundary conditions can then be found by the Newton–Raphson method. The crack extension force G can then be obtained from

$$G = \frac{1}{2\pi a}\frac{\partial}{\partial a}[\Pi + \Lambda]_{q,w_0=\mathrm{const}}, \tag{5}$$

where Π is the potential energy of the pressure forces and Λ is the strain energy of the bending and membrane stresses. Although only a comparatively few terms in the series given in Equations 4 are needed to calculate the maximum deflection of the blister to three figure accuracy, more than thirty terms are required using double precision to calculate the crack extension force to three figure accuracy up to a non-dimensional deflection of 8 because significant energy is stored in a boundary layer at the edge of the blister. The crack extension force factor Φ, is given in Figure 2 as a function of the maximum non-dimensional blister

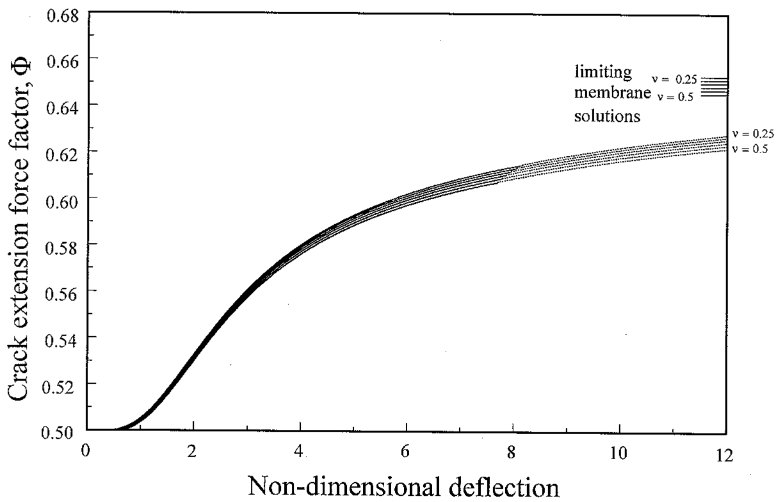

Figure 2. Crack extension force factor Φ, as a function of the blister deflection (dotted lines are extrapolations using Equation 6).

Table 2. The coefficients for the crack extension force factor Φ, for small deflections $(W_0 < 4)$

ν	a_2	a_4	a_6
0.25	8.34×10^{-3}	-1.04×10^{-4}	-7.18×10^{-6}
0.3	8.12×10^{-3}	-6.90×10^{-5}	-8.84×10^{-6}
0.35	7.87×10^{-3}	-2.89×10^{-5}	-1.07×10^{-5}
0.4	7.74×10^{-3}	-3.38×10^{-5}	-9.75×10^{-6}
0.45	7.40×10^{-3}	1.35×10^{-5}	-1.18×10^{-5}
0.5	7.01×10^{-3}	6.63×10^{-5}	-1.40×10^{-5}

deflection W_0. The program would not converge for non-dimensional defections $W_0 > 8$. For non-dimensional blister deflections $4 < W_0 < 8$, the crack extension force factor is given by

$$\Phi = \Phi_m[1 - \exp - 1.35W_0^{0.36}], \tag{6}$$

where Φ_m is the membrane crack extension force factor given in Table 1, to an accuracy better than 0.4%. The results in Figure 2 have been extended to a non-dimensional deflection of 12 using Equation 6. The crack extension force factor for non-dimensional blister deflections less than 4 are given by the polynomial

$$\Phi = 0.5 + a_2W_0^2 + a_4W_0^4 + a_6W_0^6, \tag{7}$$

to an accuracy greater than 1% where the coefficients are given in Table 2.

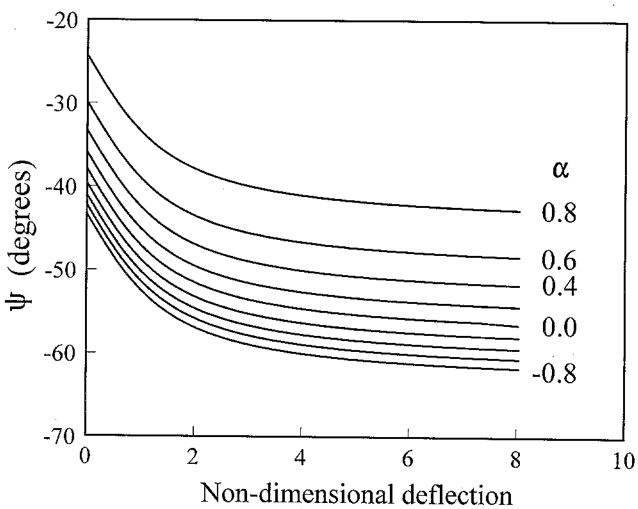

Figure 3. Mode-mixity angle ψ, as a function of the blister deflection for $\nu = 0.3$, $\beta = 0$, and $-0.8 < \alpha < 0.8$.

3. Mode-mixity in the blister test

For a strip bonded to a substrate of different elastic properties the elastic moduli dependence of the bimaterial interface system can be expressed in terms of the two Dundurs' parameters α and β given by (Dundurs, 1969)

$$\begin{aligned} \alpha &= \frac{E^* - E_s^*}{E^* + E_s^*} \\ \beta &= \frac{E^*\left(\frac{1-2\nu_s}{1-\nu_s}\right) - E_s^*\left(\frac{1-2\nu}{1-\nu}\right)}{2(E^* + E_s^*)}, \end{aligned} \tag{8}$$

for plane strain conditions, where E^* is the plane strain Young's modulus $E/(1-\nu^2)$ of the strip and the plane strain Young's modulus of the substrate E_s^*, is similarly defined and ν_s is the Poisson's ratio of the substrate. The most important of these two parameters is α. If $\beta \neq 0$ the stress intensity factors are complex and there is rapid oscillation of the stress at the crack tip which is physically unrealistic. To avoid complications, the analysis is limited to the case $\beta = 0$ either exactly or approximately and the stress intensity factors are real. Nothing is lost of the essence of the blister problem by this restriction (Hutchinson et al., 1992). Suo and Hutchinson (1990) have shown that the mode-mixity (defined by $\tan\psi = K_{\mathrm{II}}/K_{\mathrm{I}}$, where K_{I} and K_{II} are the mode I and mode II stress intensity factors) for thick substrates is given by

$$\tan\psi = \frac{K_{\mathrm{II}}}{K_{\mathrm{I}}} = \frac{\lambda\sin\omega + \cos\omega}{\lambda\cos\omega - \sin\omega}, \tag{9}$$

where $\lambda = (Nh)/(\sqrt{12}M)$ and ω is a function of α and β. The mode-mixity angles ψ, have been calculated for $\nu = 0.3, \beta = 0$ for a range of values of α and for $\alpha = \beta = 0$ for a range in values of ν, and are shown in Figures 3 and 4.

The mode-mixity analysis given above becomes inaccurate as the blister deflection increases. As the blister behaviour tends to that of a membrane, the shear stresses become of the same

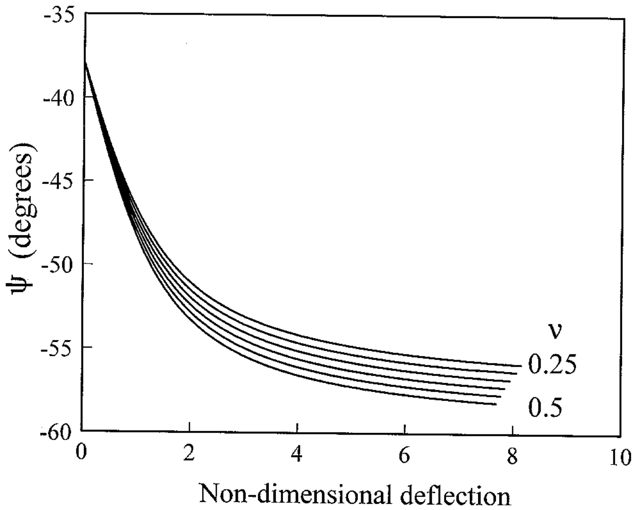

Figure 4. Mode-mixity angle as ψ, a function of the blister deflection for $\alpha = \beta = 0$, and $0.25 < \nu < 0.5$.

order as the bending stress and cannot be ignored. Also as the behaviour of the blister becomes more membrane like, though the bending and shear stresses away from the edge of the blister are of smaller order than the membrane stresses, the bending stress at the edge of the blister increases because the radius of curvature increases without limit. Hence the apparent limit solution obtained from Equation (9) that $\psi = \omega$ for a membrane is incorrect. Figures 3 and 4 suggest that the mode-mixity may not change much for $W_0 > 8$.

4. Discussion and conclusions

Although it is not apparent in Figure 2, the crack extension force is very slightly less than 0.5 for very small deflections. The smallest value of Φ occurs for $\nu = 0.5$ and is 0.4996. The difference is so slight as to make no practical difference, but it does highlight the problem of analysis because a first order perturbation solution predicts that Φ decreases with increasing deflection.

The deflection of the blister has to be quite large $W_0 > 4$, before the value of Φ is nearer to the membrane solution than the plate solution. Hence, in practical fracture tests Φ may be nearer to 0.5 than the membrane solution given in Table 1. For more accurate values Equations 6 and 7 should be used.

Equation 6 gives such a good fit to the results over the range $4 < W_0 < 8$, that it can be used accurately for $W_0 > 8$. However, if h/a is large Equation 6 will be in error for large deflections as has been discussed earlier.

Interfacial fracture energy increases with increase in the mode-mixity angle (Evans, 1990). In Figures 3 and 4 it is seen that the mode-mixity increases with non-dimensional blister deflection thus one might expect that the interfacial fracture energy in the blister test would increase if the film thickness is significantly decreased unless a/h is also decreased so that

$$\left(\frac{a}{h}\right) = \left(\frac{h}{h_0}\right)^{1/4} \left(\frac{a_0}{h_0}\right), \tag{10}$$

where a_o and h_o are the reference radius and thickness respectively. The limiting mode-mixity as the blister becomes membrane-like has not been obtained but it is suggested that it may not be far different from the values given for W_0 in Figures 3 and 4.

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