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The Root Angle in Elastoplastic Peeling Tests

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Abstract. Peel test has been widely used to assess adhesion toughness when at least one of the adherends is flexible. When adhesion is relatively strong and the adherend is thin, plastic deformation is inevitable. In an overall energy consideration, the amount of the non-essential work has to be evaluated for a correct calculation of the essential work for delamination. Previous experimental work and finite element modelling results show that there is a significant root angle at the peeling front, which affect the determination of the amount of external work that goes into the interface separation. Therefore it is crucial to correctly estimate the root angle. Most existing work on peeling angle estimation assumes the film is elastic sitting on an elastic foundation. This causes discontinuity in the deformation state of at the peeling front.

This paper presents a solution for the peeling angle and therefore the delamination toughness etc, for an elastic – linear work hardening adherend attached to a rigid substrate. A model of elastoplastic beam on foundation is applied to account for the fact that the film is not clamped at its mid-section. On the peeling front, a fracture process zone (FPZ) is placed with a simple traction – separation law. Root angle is accounted for mainly from this highly non-proportional deformation zone. The influence of the applied loading angle and the interfacial strength on the root angle has been presented.

Introduction

Peeling test is a convenient way of assessing the interfacial toughness between two components when at least one of them is flexible. When the peeled arm remains elastic, the peeling force is proportional to, and therefore a direct measurement of the adhesion toughness, R

$$R = \frac{P}{b}(1-\cos\phi), \tag{1}$$

in which P is the peeling force, b the width of the peel and ϕ the peeling angle at the far end. This solution ignores the elastic tensile deformation that is usually small compared with bending.

However when plastic deformation is present, the mechanics of peeling test is much more complicated as evidenced by the continuing effort during the past decade or so [1-11]. One important issue is to account for the non-essential work during peeling in order to correctly calculate the debonding energy, which is generally considered as a materials property. Overall energy equilibrium states that the fracture energy is equal to external work input less the amount of dissipated plastic work and stored elastic work. For steady-state peeling, the rate of this non-essential work is given by the amount of work it has experienced from the far attached end, where the curvature is zero, to the other end of the applied loading, where the curvature is returning to zero. Depending on the degree of maximum plasticity, i.e. the maximum curvature, K_m, reached during the process, the amount of non-essential work will be different. For example, if K_m is less than the elastic limit, there will be no plastic work and equation (1) applies. Over the plastic limit there is a possibility that the beam undergoes reverse plastic bending when K_m exceeds certain limit. Leaving the details to a later stage, the importance of calculating the maximum bending curvature has to be highlighted at this stage.

As illustrated in Figure 1, for the part to the right of the cracking front ($x \geq 0$), large displacement beam theory has been widely adopted [1,2,6,7]. It will be elaborated later that the maximum curvature depends the angle at the root, θ_r. Experimental evidence and finite element analysis both verified that the un-peeled adherend could not be modelled as clamped beam, which would have a zero root angle. This is because in a peeling test the bottom surface is attached to the substrate, not the neutral axis. Also there is a significant non-linear stretching in the fracture process zone (FPZ). In a pioneering work by Kim and Avaras [1,2], the significance of root angle was highlighted but they did not provide solution on how to estimated it. The first attempt to model the root angle was probably by Williams and Kinloch [3-5], where beam on elastic foundation theory was employed. In their analysis, the part for $x \geq 0$ was modelled correctly by large displacement elastoplastic beam analysis but the attached part was modelled as elastic beam only. This clearly created a discontinuity in the deformation state at $x = 0$. Work by Moidu and co-workers [9] was based on a similar approach and had been adopted by Parks et al [12-14] in their analysis on micro-electronic applications. A recent work by Wei and Hutchinson [8] tackled the problem in two parts separately: 2D finite element analysis of the beam for $x \leq L_l$, and large displacement analysis for $x \geq L_l$. In their analysis L_l was a point located to the right of peeling front where reverse plastic deformation had not yet occurred. A fracture process zone was embedded at the crack front with the traction – separation law proposed by Tvergaard and Hutchinson [15].

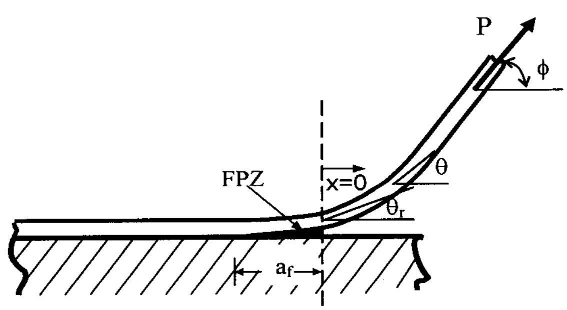

Figure 1. A model for the peeling test

The current work places an embedded fracture process zone at the tip of the delamination, coupled with beam on foundation theory for the rest of the film on substrate. The adherand is allowed to deform plastically into the region $x \leq 0$ and therefore the discontinuity in [3-5] can be eliminated. The two sources of root angle are accounted for by the beam on elastic foundation and the deformation of the FPZ respectively. In both sections ($x \geq 0$ and $x \leq 0$) analytical solutions are obtainable. Matching the solutions at the boundaries relates the root angle to other important parameters such as the maximum curvature, the strength in the FPZ, length of the FPZ and the essential work of delamination, etc.

The Mechanics of Peeling

Materials Behaviour. The following analysis considers peeling of one slender film made of elastic – linear work hardening material adhered to rigid substrate (Figure 1). Reverse bending is supposed to happen at the same yielding strength but opposite sign, see Figure 2. We assume that the beginning of the yielding of a beam occurs when the applied moment, M, causes the yielding of a plane strain beam according to Mises criterion so that the critical moment, M_e, is given by

$$M_e = \frac{\sigma_y bh^2}{6}\frac{1}{\left(1-\nu+\nu^2\right)^{\frac{1}{2}}}, \tag{2}$$

where σ_y is the uniaxial yielding stress, ν the Poisson's ratio, b and h the width and the height of the beam respectively. The corresponding bending curvature at the elastic limit is

$$K_e = \frac{\sigma_y}{Eh}\frac{2\left(1-\nu^2\right)}{\left(1-\nu+\nu^2\right)^{\frac{1}{2}}}. \tag{3}$$

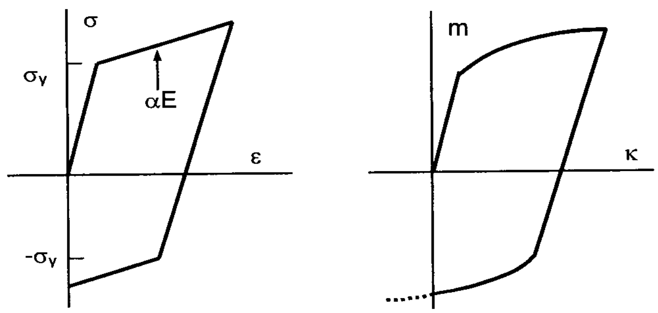

Figure 2. Left: elastic – linearly work hardening stress – strain curve; right: the corresponding bending moment – curvature relationship

As shown in Figure 2, the bending moment – curvature has the following relationships at various stages of deformation:
(1) elastic deformation, $\kappa \le 1$

$$m = \kappa \tag{4a}$$

(2) elastoplastic loading, $\kappa \ge 1$

$$m = \frac{(1-\alpha)}{2}\left(3-\frac{1}{\kappa^2}\right)+\alpha\kappa \tag{4b}$$

(3) elastoplastic deformation with linear elastic unloading, $\kappa \geq [(1-\alpha)\kappa_m - (2-\alpha)]$

$$m = \frac{(1-\alpha)}{2}\left(3 - \frac{1}{\kappa_m^{\ 2}} - 2\kappa_m\right) + \kappa \tag{4c}$$

(4) elastoplastic deformation with reverse plastic loading, $-\kappa_m \leq \kappa \leq [(1-\alpha)\kappa_m - (2-\alpha)]$

$$m = \frac{(1-\alpha)(2-\alpha)^3}{2[(1-\alpha)\kappa_m - \kappa]^2} - \alpha[(1-\alpha)\kappa_m - \kappa] - \frac{(1-\alpha)}{2\kappa_m^{\ 2}} - (1-\alpha)^2, \tag{4d}$$

where m and κ are non-dimensional moment and curvature defined as $m = \frac{M}{M_e}$ and $\kappa = \frac{K}{K_e}$ respectively, α is the linear work hardening coefficient in the stress (σ) – strain (ε) relationship $\sigma = \sigma_y + \alpha(E\varepsilon - \sigma_y)$, and κ_m is the maximum value of κ, which is always at the front of the peel.

Large Displacement Beam Bending. Without going into the details we simply apply existing theory [1,6] to our model. The maximum curvature at the peeling front is related to the applied force and the loading angle as

$$\kappa_m = [\eta(1 - \cos(\phi - \theta_r))]^{\frac{1}{2}} \tag{5a}$$

for $\kappa_m \leq \frac{2-\alpha}{1-\alpha}$ when no reverse plastic bending occurs, and

$$\begin{aligned} &\alpha[(1-\alpha)\kappa_m - (2-\alpha)]^2 \kappa_m + (1-\alpha)^2(2-\alpha)\kappa_m^{\ 2} \\ &- 2(1-\alpha)(2-\alpha)^2\kappa_m - \eta\kappa_m(1 - \cos(\phi - \theta_r)) + (2-\alpha)^3 \\ &+ (2-\alpha)(\kappa_m - 1)(\alpha\kappa_m + 2 - \alpha)\kappa_m = 0 \end{aligned} \tag{5b}$$

for $\kappa_m \geq \frac{2-\alpha}{1-\alpha}$ when reverse plastic bending does occur. In both equations (5a) and (5b) θ_r is the root angle (see Figure 1) and η is the non-dimensional load defined as $\eta = \frac{6EP}{\sigma_y^{\ 2}bh}$.

Essential Work of Debonding. The total energy released during peeling, which is equal to the essential work of debonding (or toughness), R, comprises contribution from two parts: the axial tensile energy release rate, R^t, and the one from bending, R^b. The tensile part is given by

$$R^t = \frac{P}{b}\varepsilon^t - h\int_0^{\varepsilon^t} \sigma d\varepsilon, \tag{6}$$

where P is the external peeling load, ε^t is the tensile strain, which is defined by $\frac{P}{Ebh}$.

The bending energy equilibrium requires that

$$Pdl(1-\cos\phi) = R^b bdl + dW^b, \tag{7}$$

where dW^b is the work done in bending. Usually in a peeling test the energy release rate in tension, R^t, is much small compared with the bending counterpart, R^b. But there are situations that it can not be ignored especially when large plastic deformation exists.

The total debonding toughness is therefore expressed as

$$R = \frac{P}{b}(1-\cos\phi+\varepsilon^t) - \frac{\sigma_y{}^2 h}{3E}\int_{his} m(\kappa)d\kappa - h\int_0^{\varepsilon^t}\sigma d\varepsilon. \tag{8}$$

The first term in the right hand side of equation (8) is the external work input and the second and third terms are bending and tension work expenditures respectively.

Bending Work Expenditure. When the peeling film is long enough, steady state is reached shortly after initiation. The total bending work increment for an infinitesimal peel growth, dl, is equal to the work done in this same section of material which has been bent from a straight section to the maximum curvature, and then unbent back to straight (i.e. κ = 0). The total bending work expenditure is the integral of this full history of moment – curvature relationship and its rate is

$$\frac{dW^b}{dl} = \int_{his} M(K)dK = M_e K_e \int_{his} m(\kappa)d\kappa = \frac{\sigma_y{}^2 bh}{3E}\int_{his} m(\kappa)d\kappa. \tag{9}$$

The importance of the integral $\int_{his} m(\kappa)d\kappa$ is clear from Equation (9). If the peak curvature, κ_m, during peeling is known, the full history of work expenditure of bending and un-bending a section of material is given by

$$\int_{his} m d\kappa = 0 \qquad \text{for } \kappa_m \le 1 \tag{10a}$$

$$\int_{his} m d\kappa = \frac{(1-\alpha)}{2}\left(\frac{2}{\kappa_m} + \kappa_m{}^2 - 3\right) \qquad \text{for } 1 \le \kappa_m \le \frac{2-\alpha}{1-\alpha} \tag{10b}$$

$$\int_{his} m d\kappa = \frac{1}{2}\{(1-\alpha)[\alpha(2-\alpha)\kappa_m{}^2 + 3(1-\alpha)(2-\alpha)\kappa_m - 3(5-4\alpha+\alpha^2)] + [(2-\alpha)^3 + 2(1-\alpha)]/\kappa_m\} \qquad \text{for } \kappa_m \ge \frac{2-\alpha}{1-\alpha}. \tag{10c}$$

Now the task is to determine the degree of plasticity that different equations in (10) would apply to find the integral $\int_{his} m d\kappa$. Once this is found out, it can be put into equation (8) to calculate R. Up to this point, if there is no root rotation, i.e. $\theta_r = 0$, the above analysis is sufficient for calculating the delamination toughness. However since in reality θ_r is non-zero, it has to be found out first. The following analysis aims at solving θ_r by working on the attached part ($x \le 0$).

Solution for the Adhered Film ($x \le 0$). For the part $x \le 0$, the film is treated as elastoplastic, sitting on the FPZ ($-a_f \le x \le 0$) and the elastic foundation ($x \le -a_f$), see Figure 1. Instead of the cohesive law by Tvergaard and Hutchinson [15], current work adopts a simpler tension stress – displacement relationship with a uniform stress for crack opening displacements up to separation. This simplification is justifiable because the root angle is mainly controlled by the opening stress in the FPZ. The governing equation for these two sub-sections is given by

$$\begin{cases} E'I\dfrac{d^4y(x)}{dx^4} + b\sigma_f = 0 & \text{for} -a_f \le x \le 0 \\ E'I\dfrac{d^4y(x)}{dx^4} + ky(x) = 0 & \text{for } x \le -a_f \end{cases}, \quad (11)$$

in which σ_f is the tension stress in the FPZ, E' the plane strain modulus, $E/(1-\nu^2)$, I the second moment of area, y(x) the vertical displacement and k the modulus of foundation. It has been proved [16,17] that the increased compliance due to the fact that the bottom surface is clamped to the substrate can be quite accurately estimated if the foundation is taken as half the film thickness. The modulus of the elastic foundation is given by [18]

$$k = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\left(\frac{2b}{h}\right). \quad (12)$$

The root angle can not be solved solely by analysing the attached part since it depends on the applied moment at the $x = 0$, which in turn varies with the root angle from Equation (4) and (5). Thus the solutions for $x \le 0$ and $x \ge 0$ have to be coupled together. By matching the load, displacement and the slope at the root ($x = 0$) and the end of FPZ ($x = -a_f$), numerical iteration will determine a proper root angle that satisfy all the equations. This scheme enables us bring together all the important features for a peeling test, such as the θ_r, κ_m, a_f, the critical opening displacement at the peeling front, δ, and the interfacial strength in the FPZ, σ_f. If any one of these parameters is known, the rest can be solved.

Discussion and Conclusion

The importance of root angle now becomes clear. In certain sense, it is not only directly controlling the value of κ_m, which in turn determines the debonding toughness of peeling, it also reveals other important intrinsic features of delamination impossible to be revealed before. Although perhaps burdensome, it is possible to directly measure the root angle as in the work by Kinloch et al [4,5]. A previous work by the author [19] verified that, by fitting the calculated root angle to the experimentally observed average value in Kinloch's work [4,5], similar adhesion toughness was obtained. These values did not vary much with the peeling angle [19]. This seems to suggest that the energy consumed for the peeling is not much influenced by the applied peeling angle. However the interfacial strength was found to be quite different for various loading angles [19]. This piece of information was not available in [4,5].

Suppose the debonding energy is independent of the peeling angle, Figure 3 illustrates the effect of loading angle and interfacial strength on the value of the root angle. Calculation is based on a "model" PET-like material with $E = 4$ GPa, $\sigma_y = 100$ MPa, $\nu = 0.4$, $R = 100$ J/m^2. Linear work hardening co-efficient is taken as $\alpha = 0.05$. Film thickness is chosen to be 10 times the characteristic length, r_y, given by

$$r_y = \frac{ER}{3\pi(1-\nu^2)\sigma_y^2}. \tag{13}$$

From Figure 3 the root angle increases with the loading angle. This is consistent with the observation by Kinloch et al [4,5] and Wei and Hutchinson's [8] theoretical analysis. With larger interfacial strength, the root angle decreases for the same loading angle, ϕ. The decreasing rate is faster when σ_f is lower. This latter result shows a different trend with the one of crack tip opening angle by Wei and Hutchinson [8], where the angle is increasing with interfacial strength. Despite the fact that the angles are defined differently in the two studies, the difference is only expected to be in the values. More work has to be done in the future to find out the discrepancy. In current author's view, the higher the σ_f, the "stiffer" the fracture process zone and therefore the angle should open less.

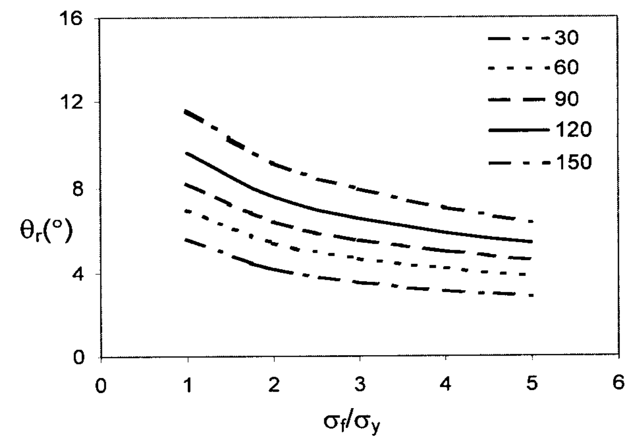

Figure 3. Root angle θ_r at the peeling root as a function of the normalized interfacial strength at different loading angles with $h/r_y = 10$.

This paper presents a complete analysis for elastoplastic peeling of an elastic – linearly work hardening material. Emphasis is laid on the role of root angle and the factors influencing this angle. Through numerical iteration, the current approach is able to correlate all the important parameters characterizing a peeling test, such as the root angle, the maximum curvature, fracture process zone size & strength, critical crack tip opening displacement, and the interfacial toughness. If any one of these parameters is known, the rest can be solved. Among these parameters the root angle may be the only one that can be measured directly, all the others have to be, and can be calculated by current approach.

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