Multiple Trajectory Search for Large Scale Global Optimization

Lin-Yu Tseng and Chun Chen

Abstract—In this paper, the multiple trajectory search (MTS) is presented for large scale global optimization. The MTS uses multiple agents to search the solution space concurrently. Each agent does an iterated local search using one of three candidate local search methods. By choosing a local search method that best fits the landscape of a solution’s neighborhood, an agent may find its way to a local optimum or the global optimum. We applied the MTS to the seven benchmark problems designed for the CEC 2008 Special Session and Competition on Large Scale Global Optimization.

I. INTRODUCTION

Global numerical optimization is an important research issue because many real-life problems can be formulated as global numerical optimization. Many of numerical optimization problems cannot be solved analytically, and consequently, numerical algorithms were proposed to solve these problems. Many of these algorithms are evolutionary algorithms. In CEC 2005, a special session of real-parameter optimization had been organized. Several algorithms [1][4][5][6][11] were presented in this special session and all these algorithms were tested on a suite of 25 benchmark functions with dimensions 10 and 30. Besides these algorithms, some of evolutionary algorithms proposed recently are briefly surveyed in the following. Kazarlis et al. [2] combined a standard GA with a microgenetic algorithm (MGA) [GA with small population and short evolution] to solve the numerical optimization problem. In their method, the MGA operator fits the landscape of a solution’s neighborhood, an agent may find its way to a local optimum or the global optimum. We applied the MTS to the seven benchmark problems designed for the CEC 2008 Special Session and Competition on Large Scale Global Optimization [7], the experimental results reveal that the MTS is effective and efficient in solving these problems. The remainder of this paper is organized as follows. Section II gives some definitions. Section III describes the MTS algorithm. Section VI presents the experimental results and Section IV draws the conclusion.

II. PRELIMINARIES

A. Problem Definition

The global numerical optimization problem is defined as follows.

Minimize $F(X)$

Subject to $l \leq X \leq u$

where $X=(x_1, x_2, \ldots, x_N)$ is a variable vector in $R^N$ space and $F(X)$ is the objective function. $l=(l_1, l_2, \ldots, l_N)$ and $u=(u_1, u_2, \ldots, u_N)$ define the feasible solution space, that is, a feasible solution $X=(x_1, x_2, \ldots, x_N)$ must satisfies $l_i \leq x_i \leq u_i$ for $i=1, 2, \ldots, N$. The feasible solution space is denote by $[l, u]$.

B. Orthogonal Array and Simulated Orthogonal Array

We briefly introduce the concept of orthogonal arrays which are used in experimental design methods. Suppose in an experiment, there are $k$ factors and each factor has $q$ levels. In order to find the best setting of each factor’s level, $q^k$ experiments must be done. Very often, it is not possible or cost effective to test all $q^k$ combinations. It is desirable to sample a small but representative sample of combinations for testing. The orthogonal arrays were developed for this purpose. An orthogonal array $OA(n,k,q,t)$ is an array with $n$ rows and $k$ columns which is a representative sample of $n$ testing representative of a stochastic region described by a multivariate Gaussian distribution. Zhong et al. [12] integrated multiagent systems and genetic algorithms to form a new algorithm called the multiagent genetic algorithm (MAGA) for numerical optimization. All above mentioned algorithms except the last one were tested on functions with dimensions less than or equal to 100. The last algorithm (MAGA) was tested on functions with dimensions from 20 to 10,000.

In this paper, the multiple trajectory search (MTS) was presented to solve the large scale global optimization problem. The MTS had been used to solve the multi-objective optimization problems and obtained satisfactory results [9]. We used the same framework but improved the local search methods for numerical optimization. When applying the MTS to the seven benchmark problems designed for the CES 2008 Special Session and Competition on Large Scale Global Optimization [7], the experimental results reveal that the MTS is effective and efficient in solving these problems.

The remainder of this paper is organized as follows. Section II gives some definitions. Section III describes the MTS algorithm. Section VI presents the experimental results and Section IV draws the conclusion.

Lin-Yu Tseng is with the Institute of Networking and Multimedia and the Department of Computer Science and Engineering, National Chung Hsing University, 250 Kuo Kuang Road, Taichung, Taiwan 402, ROC (corresponding author; phone: 866-4-22874020; e-mail: lytseng@cs.nchu.edu.tw).

Chun Chen is a PhD student with the Department of Computer Science and Engineering, National Chung Hsing University, 250 Kuo Kuang Road, Taichung, Taiwan 402, ROC (e-mail: phd9401@cs.nchu.edu.tw).

978-1-4244-1823-7/08/$25.00 ©2008 IEEE
experiments that satisfies the following three conditions. (1) For the factor in any column, every level occurs the same number of times. (2) For the t factors in any t columns, every combination of q levels occurs the same number of times. (3) The selected combinations are uniformly distributed over the whole space of all the possible combinations. In the notation OA(n,k,q,t), n is the number of experiments, k is the number of factors, q is the number of levels of each factor and t is called the strength.

The orthogonal arrays exist for only some specific n’s and k’s. So it is not appropriate to use the OA in some applications. We proposed the simulated OA (SOA) in this study. The SOA satisfies only the first of the above mentioned three conditions, but it is easy to construct an SOA of almost any size.

Suppose there are k factors and each factor has q levels, an m×k simulated orthogonal array SOA(m×k), m is a number of times. (2) For the t factors in any t columns, every level occurs the same number of times. (3) For the factor in any column, every level occurs the same number of times. (4) The selected combinations are uniformly distributed over the feasible solution space. The initial SOA tends to make these M initial solutions uniformly distributed over the whole space of all the possible combinations. In the notation OA(n,k,q,t), n is the number of experiments, k is the number of factors, q is the number of levels of each factor and t is called the strength.

The selected combinations are uniformly distributed over the whole space of all the possible combinations. In the notation OA(n,k,q,t), n is the number of experiments, k is the number of factors, q is the number of levels of each factor and t is called the strength.

The multiple trajectory search algorithm is described in the following.

### III. MULTIPLE TRAJECTORY SEARCH

In this section, we present the multiple trajectory search (MTS) for the large scale global optimization problem. In the beginning, the MTS generates M initial solutions by utilizing the simulated orthogonal array SOA(m×k), where the number of factors corresponds to the dimension N and the number of levels of each factor is taken to be M. So each of 0, 1, …, M-1 will appear once in every column. Using SOA tends to make these M initial solutions uniformly distributed over the feasible solution space. The initial search range for local search methods is set to half of the difference between the upper bound and the lower bound. Afterwards, local search methods will change the search range.

The MTS consists of iterations of local searches until the maximum number of function evaluations is reached. In the first iteration, the MTS conducts local searches on all of M initial solutions. But in the following iterations, only some better solutions are chosen as foreground solutions and the MTS conducts local searches on these solutions. Three local search methods are provided for the MTS. The MTS will first test the performance of these local search methods and then choose the one that performs best, that is, the one that best fits the landscape of the neighborhood of the solution, to do the search. After conducting the search on foreground solutions, the MTS applies Local Search 1 to the current best solution trying to improve the current best solution. Before the end of an iteration, some better solutions are chosen as the foreground solutions for the next iteration. The multiple trajectory search algorithm is described in the following.

### Multiple Trajectory Search

/*Generate M initial solutions */
Build simulated orthogonal array SOA

For i = 1 to M
    For j = 1 to N
        \[ X[i][j] = \bar{i} + (\alpha_i - 1) \times \text{SOA}[i, j] / (M-1) \]
    End For
End For

Evaluate function values of Xi’s

For i = 1 to M
    If Enable[i] = TRUE
        Improve[i] = TRUE
    End If
End For

While (#ofEvaluation \leq \text{predefined_max_evaluation})
    For i = 1 to M
        If Enable[i] = TRUE
            Then
                GradeXi = 0
                LS1_TestGrade = 0
                LS2_TestGrade = 0
                LS3_TestGrade = 0
                For j = 1 to #ofLocalSearchTest
                    LS1_TestGrade = LS1_TestGrade + LocalSearch1(Xi, SearchRangeXi)
                    LS2_TestGrade = LS2_TestGrade + LocalSearch2(Xi, SearchRangeXi)
                    LS3_TestGrade = LS3_TestGrade + LocalSearch3(Xi, SearchRangeXi)
                End For
                Choose the one with the best TestGrade and let it be LocalSearchK /* K may be 1, 2, or 3 */
                For j = 1 to #ofLocalSearch
                    GradeXi = GradeXi + LocalSearchK(Xi, SearchRangeXi)
                End For
            End If
        End For
    End While

In the MTS, three local search methods are used for searching different landscape of the neighborhood of a solution.

Local Search 1 searches along one dimension from the first dimension to the last dimension. Local Search 2 is similar to Local Search 1 except that it searches along about one-fourth of dimensions. In both local search methods, the search range (SR) will be cut to one-half until it is less than \(1 \times 10^{-15}\) if the
Search 1 and Local Search 2 are listed in the following and the search proceeds to consider the next dimension. If it is not, the solution is restored and then the solution’s coordinate of this dimension is added by 0.5*SR, again to see if the objective function value is improved. If it is, the search proceeds to consider the next dimension. Local Search 1, on the dimension concerning the search, the previous local search does not make improvement. In Local Search 1, on the dimension concerning the search, the solution’s coordinate of this dimension is first subtracted by SR to see if the objective function value is improved. If it is, the search proceeds to consider the next dimension. If it is not, the solution is restored and then the solution’s coordinate of this dimension is added by 0.5*SR, again to see if the objective function value is improved. If it is, the search proceeds to consider the next dimension. If it is not, the solution is restored and then the search proceeds to consider the next dimension. 

Local Search 1 and Local Search 2 are listed in the following.

Function LocalSearch1(Xk, SR)

If Improve[k] = FALSE
    Then SR = SR/2
    If SR < 1e-15
        Then SR = (UPPER_BOUND-LOWER_BOUND) * 0.4
    End If
End If

Improve[k] = FALSE
For i = 1 to N
    Xk[i] ← Xk[i]- SR
    If Xk is better than current best solution
        Then grade ← grade + BONUS1
            Update current best solution
    End If
End If
If function value of Xk is the same
    Then restore Xk to its original value
Else
    If function value of Xk degenerates
        Then restore Xk to its original value
            Xk[i] ← Xk[i] + 0.5* SR
            If Xk is better than current best solution
                Then grade ← grade + BONUS1
                    Update current best solution
            End If
        If function value of Xk has not been improved
            Then restore Xk to its original value
        Else
            grade ← grade + BONUS2
            Improve[k] ← TRUE
        End If
    Else
        grade ← grade + BONUS2
        Improve[k] ← TRUE
    End If
End If
End If
End For

return grade

Function LocalSearch2(Xk, SR)

If Improve[k] = FALSE
    Then SR = SR/2
    If SR < 1e-15
        Then SR = (UPPER_BOUND-LOWER_BOUND) * 0.4
    End If
End If
End If
End If

Improve[k] ← FALSE
For l = 1 to N
    For r[i] ← Random{0,1,2,3}
        Then Xk[i] ← Xk[i] - 0.5*SR*D[i]
    End For
End If
End For
If Xk is better than current best solution
    Then grade ← grade + BONUS1
        Update current best solution
End If
If function value of Xk is the same
    Then restore Xk to its original value
Else
    if function value of Xk degenerates
        Then restore Xk to its original value
    End If
    If r[i] = 0
        Then Xk[i] ← Xk[i] + 0.5*SR*D[i]
    End If
End If
End If
If Xk is better than current best solution
    Then grade ← grade + BONUS1
        Update current best solution
End If
If function value of Xk has not been improved
    Then restore Xk to its original value
Else
    grade ← grade + BONUS2
    Improve[k] ← TRUE
End If
End If
Else
    grade ← grade + BONUS2
    Improve[k] ← TRUE
End If
End If
End For
return grade

Local Search 3 is different from Local Search 1 and Local Search 2. Local Search 3 considers three small movements along each dimension and heuristically determines the movement of the solution along each dimension. In Local Search 3, although the search is along each dimension from the first dimension to the last dimension, the evaluation of the objective function value is done after searching all the dimensions, and the solution will be moved to the new position only if the objective function has been improved at this evaluation. Local Search 3 is described in the following.

Function LocalSearch3(X, SR)
A. PC Configuration

System: Windows XP RAM: 1GB

B. Parameter Setting

M=5 #ofForeground=3 #ofLocalSearchTest=3 #ofLocalSearch=100
#ofLocalSearchBest=150 BONUS1=10 BONUS2=1 a=Random[0.4, 0.5], b=Random[0.1, 0.3], c=Random[0, 1],

C. Experimental Results

The function error value (f(x)-f(x*)) after FES/100, FES/10, and Max_FES are listed in Table 1, 2 and 3 respectively for seven problems with dimensions 100, 500 and 1000. Seven benchmark problems are listed in Table 4. The details can be found in [7]. The convergence graphs for problems 1-6 and problem 7 with dimension D=1000 are depicted in Figure 1 and Figure 2 respectively.

V. CONCLUSIONS

The multiple trajectory search (MTS) was presented and applied to solve the seven benchmark problems provided for the purpose of competition in CEC 2008. The seven problems are with dimensions 100, 500 and 1000, so the scalability of algorithms can also be tested. When MTS searches the neighborhood of a solution, it tests the performance of the three predefined local search methods first and then chooses the best one to conduct the local search. Therefore, the MTS performs its search automatically adapting to the landscape of the solution’s neighborhood. For problems 1-6 whose optimal solutions are already known, the MTS can find the optimal solutions of four problems with FES/10 when dimension is 100. Also, it can find the optimal solutions of two and three problems with FES/10 when dimension is 500 and 1000 respectively.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the support of National Science Council of ROC under the contract NSC 96-2628-E-005-074-MY3.

REFERENCES


<table>
<thead>
<tr>
<th>µ</th>
<th>Evaluation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00e+3</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;(best)</td>
<td>2.7647e+03</td>
<td>4.5091e+01</td>
<td>1.4427e+01</td>
<td>3.0150e+02</td>
<td>6.2602e+01</td>
<td>7.1666e+00</td>
<td>-1.3010e+03</td>
</tr>
<tr>
<td></td>
<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
<td>1.1054e+04</td>
<td>4.9549e+01</td>
<td>1.4704e+01</td>
<td>3.6804e+02</td>
<td>1.5153e+02</td>
<td>1.2654e+01</td>
<td>-1.2865e+03</td>
</tr>
<tr>
<td></td>
<td>13&lt;sup&gt;th&lt;/sup&gt;(median)</td>
<td>1.5884e+04</td>
<td>5.0089e+01</td>
<td>1.7724e+01</td>
<td>4.0088e+02</td>
<td>1.7169e+02</td>
<td>1.3316e+01</td>
<td>-1.2730e+03</td>
</tr>
<tr>
<td></td>
<td>19&lt;sup&gt;th&lt;/sup&gt;</td>
<td>1.8170e+04</td>
<td>5.0787e+01</td>
<td>4.8272e+02</td>
<td>4.6015e+02</td>
<td>1.8816e+02</td>
<td>1.3792e+01</td>
<td>-1.2681e+03</td>
</tr>
<tr>
<td></td>
<td>25&lt;sup&gt;th&lt;/sup&gt;(worst)</td>
<td>2.4348e+04</td>
<td>6.1540e+01</td>
<td>1.1203e+03</td>
<td>5.2764e+02</td>
<td>2.2808e+02</td>
<td>1.4581e+01</td>
<td>-1.2598e+03</td>
</tr>
<tr>
<td>Mean</td>
<td>1.4326e+04</td>
<td>5.0646e+01</td>
<td>3.1953e+02</td>
<td>4.1221e+02</td>
<td>1.6219e+02</td>
<td>1.2776e+01</td>
<td>-1.2772e+03</td>
<td></td>
</tr>
<tr>
<td>Std</td>
<td>5.6623e+03</td>
<td>3.1603e+00</td>
<td>4.5086e+02</td>
<td>6.2943e+01</td>
<td>4.3027e+01</td>
<td>1.6268e+00</td>
<td>7.3752e+00</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>µ</th>
<th>Evaluation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00e+4</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;(best)</td>
<td>0.0000e+00</td>
<td>2.1902e-02</td>
<td>1.1369e-13</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4866e+03</td>
</tr>
<tr>
<td></td>
<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
<td>0.0000e+00</td>
<td>2.6218e-02</td>
<td>1.6596e-06</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4739e+03</td>
</tr>
<tr>
<td></td>
<td>13&lt;sup&gt;th&lt;/sup&gt;(median)</td>
<td>0.0000e+00</td>
<td>4.0085e-02</td>
<td>1.9553e-06</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4686e+03</td>
</tr>
<tr>
<td></td>
<td>19&lt;sup&gt;th&lt;/sup&gt;</td>
<td>0.0000e+00</td>
<td>5.6363e-02</td>
<td>2.3313e-06</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4646e+03</td>
</tr>
<tr>
<td></td>
<td>25&lt;sup&gt;th&lt;/sup&gt;(worst)</td>
<td>0.0000e+00</td>
<td>8.9314e-02</td>
<td>1.4180e-04</td>
<td>5.5161e-10</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4567e+03</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0000e+00</td>
<td>4.3509e-02</td>
<td>9.7317e-06</td>
<td>2.2064e-11</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4695e+03</td>
<td></td>
</tr>
<tr>
<td>Std</td>
<td>0.0000e+00</td>
<td>1.8510e-02</td>
<td>2.8397e-05</td>
<td>1.1032e-10</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>4.5071e+00</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>µ</th>
<th>Evaluation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00e+5</td>
<td>1&lt;sup&gt;st&lt;/sup&gt;(best)</td>
<td>0.0000e+00</td>
<td>3.9790e-13</td>
<td>5.6843e-14</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4992e+03</td>
</tr>
<tr>
<td></td>
<td>7&lt;sup&gt;th&lt;/sup&gt;</td>
<td>0.0000e+00</td>
<td>1.9327e-12</td>
<td>5.8043e-09</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4941e+03</td>
</tr>
<tr>
<td></td>
<td>13&lt;sup&gt;th&lt;/sup&gt;(median)</td>
<td>0.0000e+00</td>
<td>5.5707e-12</td>
<td>9.0498e-09</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4872e+03</td>
</tr>
<tr>
<td></td>
<td>19&lt;sup&gt;th&lt;/sup&gt;</td>
<td>0.0000e+00</td>
<td>1.7167e-11</td>
<td>1.5000e-08</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4801e+03</td>
</tr>
<tr>
<td></td>
<td>25&lt;sup&gt;th&lt;/sup&gt;(worst)</td>
<td>0.0000e+00</td>
<td>4.7976e-11</td>
<td>7.8752e-07</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4743e+03</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0000e+00</td>
<td>1.4406e-11</td>
<td>5.1707e-08</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>-1.4861e+03</td>
<td></td>
</tr>
<tr>
<td>Std</td>
<td>0.0000e+00</td>
<td>1.8685e-11</td>
<td>1.6085e-07</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>0.0000e+00</td>
<td>5.7916e+00</td>
<td></td>
</tr>
</tbody>
</table>
Table 2 Error values achieved for problems 1-6 and function values for problem 7, with D=500

<table>
<thead>
<tr>
<th>FES</th>
<th>1&lt;sup&gt;st&lt;/sup&gt;(best)</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt;</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt;</th>
<th>4&lt;sup&gt;th&lt;/sup&gt;</th>
<th>5&lt;sup&gt;th&lt;/sup&gt;</th>
<th>6&lt;sup&gt;th&lt;/sup&gt;</th>
<th>7&lt;sup&gt;th&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50e+4</td>
<td>8.6438E+04</td>
<td>8.9075E+01</td>
<td>2.8251E+01</td>
<td>2.2379E+03</td>
<td>1.3709E+01</td>
<td>1.3532E+01</td>
<td>-6.0937E+03</td>
</tr>
<tr>
<td>13&lt;sup&gt;th&lt;/sup&gt;(median)</td>
<td>9.2725E+04</td>
<td>8.9382E+01</td>
<td>1.2535E+01</td>
<td>2.3095E+03</td>
<td>1.8958E+03</td>
<td>1.3861E+03</td>
<td>-6.0228E+03</td>
</tr>
<tr>
<td>19&lt;sup&gt;th&lt;/sup&gt;</td>
<td>1.0495E+05</td>
<td>9.0030E+01</td>
<td>1.8252E+01</td>
<td>2.3758E+03</td>
<td>1.9609E+03</td>
<td>1.4062E+03</td>
<td>-5.9951E+03</td>
</tr>
<tr>
<td>25&lt;sup&gt;th&lt;/sup&gt;(worst)</td>
<td>1.2322E+05</td>
<td>1.0699E+02</td>
<td>1.9294E+02</td>
<td>2.4678E+03</td>
<td>2.1973E+03</td>
<td>1.4400E+03</td>
<td>-5.9623E+03</td>
</tr>
<tr>
<td>Mean</td>
<td>9.9562E+04</td>
<td>9.0456E+01</td>
<td>2.3310E+01</td>
<td>2.3658E+03</td>
<td>1.9103E+03</td>
<td>1.4027E+03</td>
<td>-6.0029E+03</td>
</tr>
<tr>
<td>Std</td>
<td>1.0638E+04</td>
<td>3.4691E+00</td>
<td>3.7387E+01</td>
<td>6.6189E+01</td>
<td>4.1064E+02</td>
<td>2.1631E+01</td>
<td>1.7398E+01</td>
</tr>
</tbody>
</table>

Table 3 Error values achieved for problems 1-6 and function values for problem 7, with D=1000

<table>
<thead>
<tr>
<th>FES</th>
<th>1&lt;sup&gt;st&lt;/sup&gt;(best)</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt;</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt;</th>
<th>4&lt;sup&gt;th&lt;/sup&gt;</th>
<th>5&lt;sup&gt;th&lt;/sup&gt;</th>
<th>6&lt;sup&gt;th&lt;/sup&gt;</th>
<th>7&lt;sup&gt;th&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00e+4</td>
<td>8.6438E+04</td>
<td>8.9075E+01</td>
<td>2.8251E+01</td>
<td>2.2379E+03</td>
<td>1.3709E+01</td>
<td>1.3532E+01</td>
<td>-6.0937E+03</td>
</tr>
<tr>
<td>13&lt;sup&gt;th&lt;/sup&gt;(median)</td>
<td>9.2725E+04</td>
<td>8.9382E+01</td>
<td>1.2535E+01</td>
<td>2.3095E+03</td>
<td>1.8958E+03</td>
<td>1.3861E+03</td>
<td>-6.0228E+03</td>
</tr>
<tr>
<td>19&lt;sup&gt;th&lt;/sup&gt;</td>
<td>1.0495E+05</td>
<td>9.0030E+01</td>
<td>1.8252E+01</td>
<td>2.3758E+03</td>
<td>1.9609E+03</td>
<td>1.4062E+03</td>
<td>-5.9951E+03</td>
</tr>
<tr>
<td>25&lt;sup&gt;th&lt;/sup&gt;(worst)</td>
<td>1.2322E+05</td>
<td>1.0699E+02</td>
<td>1.9294E+02</td>
<td>2.4678E+03</td>
<td>2.1973E+03</td>
<td>1.4400E+03</td>
<td>-5.9623E+03</td>
</tr>
<tr>
<td>Mean</td>
<td>9.9562E+04</td>
<td>9.0456E+01</td>
<td>2.3310E+01</td>
<td>2.3658E+03</td>
<td>1.9103E+03</td>
<td>1.4027E+03</td>
<td>-6.0029E+03</td>
</tr>
<tr>
<td>Std</td>
<td>1.0638E+04</td>
<td>3.4691E+00</td>
<td>3.7387E+01</td>
<td>6.6189E+01</td>
<td>4.1064E+02</td>
<td>2.1631E+01</td>
<td>1.7398E+01</td>
</tr>
</tbody>
</table>

**2008 IEEE Congress on Evolutionary Computation (CEC 2008)**
### Table 4 Benchmark problems

<table>
<thead>
<tr>
<th>Benchmark problems</th>
<th>Search range</th>
<th>Optimal solution value F(x*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1(x) = \sum_{i=1}^{D} z_i + f_{bias_1}, z = x-o$</td>
<td>[-100,100]</td>
<td>-450</td>
</tr>
<tr>
<td>$o = [o_1, o_2, ..., o_D]$: the shifted global optimum.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_2(x) = \max_{i} {</td>
<td>z_i</td>
<td>, 1 \leq i \leq D} + f_{bias_2}, z = x-o$</td>
</tr>
<tr>
<td>$o = [o_1, o_2, ..., o_D]$: the shifted global optimum.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_3(x) = \sum_{i=1}^{D} \left(100(z_i^2 - z_{i+1}^2)^2 + (z_i - 1)^2\right) + f_{bias_1}, z = x-o+1$</td>
<td>[-100,100]</td>
<td>390</td>
</tr>
<tr>
<td>$o = [o_1, o_2, ..., o_D]$: the shifted global optimum.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_4(x) = \sum_{i=1}^{D} (z_i^2 - 10\cos(2\pi z_i) + 10) + f_{bias_4}, z = x-o$</td>
<td>[-5,5]</td>
<td>-330</td>
</tr>
<tr>
<td>$o = [o_1, o_2, ..., o_D]$: the shifted global optimum.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_5(x) = \frac{\sum_{i=1}^{D} z_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 + f_{bias_5}, z = x-o$</td>
<td>[-600,600]</td>
<td>-180</td>
</tr>
<tr>
<td>$o = [o_1, o_2, ..., o_D]$: the shifted global optimum.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_6(x) = -20\exp(-0.2 \sqrt{\frac{1}{D} \sum_{i=1}^{D} z_i^2}) - \exp\left(\frac{1}{D} \sum_{i=1}^{D} \cos(2\pi z_i)\right) + 20 + f_{bias_6}$</td>
<td>[-32.32]</td>
<td>-140</td>
</tr>
<tr>
<td>$z = x-o, o = [o_1, o_2, ..., o_D]$: the shifted global optimum.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_7(x) = \sum_{i=1}^{D} \text{fractal} 1D(x_i + \text{twist}(x_{(i \mod \nu)}))$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{twist}(y) = 4(y^4 - 2y^3 + y^2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{fractal} 1D(x) = \frac{3}{2^{l-1}} \sum_{i=1}^{\nu} \sum_{k=1}^{\nu} \sum_{l=1}^{\nu} \text{doubledip}(x, \text{ranl}(o), \frac{1}{2^{k-1}(2 - \text{ranl}(o)})$</td>
<td>[-1.1]</td>
<td>unknown</td>
</tr>
<tr>
<td>$\text{doubledip}(x, c, s) = \begin{cases} (-644(x-c)^6 + 3088(x-c)^4 - 392(x-c)^2 + 1)x, -0.5 &lt; x &lt; 0.5 \ 0, \text{otherwise} \end{cases}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{ranl}(o)$: double, pseudorand only chosen, with seed o, with equal probability from the interval [0,1].</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{ran2(o)}$: integer, pseudorand only choose, with seed o, with equal probability from the set {0,1,2}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1 Convergence graphs for problems 1-6 with dimension $D = 1000$.

Figure 2 Convergence graph for problem 7 with dimension $D = 1000$. 